19.56. Model: For the closed cycle of the heat engine, process $1 \to 2$ is isochoric, process $2 \to 3$ is adiabatic, and process $3 \to 1$ is isothermal. For a diatomic gas $C_V = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$.

Visualize: Please refer to Figure P19.56.

Solve: **(a)** The pressure p_2 lies on the adiabat from $2 \to 3$. We can find the pressure as follows:

$$p_2 V_2^\gamma = p_3 V_3^\gamma \Rightarrow p_2 = p_3 \left(\frac{V_3}{V_2}\right)^\gamma = \left(1.0 \times 10^5 \text{ Pa}\right)\left(\frac{4000 \text{ cm}^3}{1000 \text{ cm}^3}\right)^{7/5} = 6.964 \times 10^5 \text{ Pa}$$

The temperature T_2 can be obtained from the ideal-gas equation relating points 1 and 2:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = T_1 \frac{p_2}{p_1}\frac{V_2}{V_1} = (300 \text{ K})\left(\frac{6.964 \times 10^5 \text{ Pa}}{4.0 \times 10^5 \text{ Pa}}\right)(1) = 522.3 \text{ K}$$

(b) The number of moles of the gas is

$$R = \frac{p_1 V_1}{R T_1} = \frac{\left(4.0 \times 10^5 \text{ Pa}\right)\left(1.0 \times 10^{-3} \text{ m}^3\right)}{(8.31 \text{ J / mol K})(300 \text{ K})} = 0.1604 \text{ mol}$$

For isochoric process $1 \to 2$, $W_s = 0$ J and

$$Q = \Delta E_{th} = nC_V \Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = 741.1 \text{ J}$$

For adiabatic process $2 \to 3$, $Q = 0$ J and

$$\Delta E_{th} = nC_V \Delta T = n\left(\tfrac{5}{2}R\right)\left(T_3 - T_2\right) = -741.1 \text{ J}$$

Using the first law of thermodynamics, $\Delta E_{th} = \Delta W_s + Q$, which means $W_s = -\Delta E_{th} = +741.1$ J. W_s can also be determined from

$$W_s = \frac{p_3 V_3 - p_2 V_2}{1-\gamma} = \frac{nR\left(T_3 - T_2\right)}{1-\gamma} = \frac{\left(\frac{4}{3} \text{ J / K}\right)(300 \text{ K} - 522.3 \text{ K})}{\left(-\frac{2}{5}\right)} = 741.1 \text{ J}$$

For isothermal process $3 \to 1$, $\Delta E_{th} = 0$ J and

$$W_s = nRT_1 \ln\frac{V_1}{V_3} = -554.5 \text{ J}$$

Using the first law of thermodynamics, $\Delta E_{th} = -W_s + Q$, $Q = W_s = -554.5$ J.

	ΔE_{th} (J)	W_s (J)	Q (J)
$1 \to 2$	741.1	0	741.1
$2 \to 3$	–741.1	741.1	0
$3 \to 1$	0	–554.5	–554.5
Net	0	186.6	186.6

(c) The work per cycle is 186.6 J and the thermal efficiency is

$$\eta = \frac{W_s}{Q_H} = \frac{186.6 \text{ J}}{741.1 \text{ J}} = 0.252 = 25.2\%$$